This book is intended as an overview of a research area that combines geometries for groups (such as Tits buildings and generalizations), topological aspects of simplicial complexes from $p$-$q$-subgroups of a group (in the spirit of Brown, Quillen, and Webb), and combinatorics of partially ordered sets. The material is intended to serve as an advanced graduate-level text and partly as a general reference on the research area. The treatment offers optional tracks for the reader interested in buildings, geometries for sporadic simple groups, and $G$-$q$-equivariant homology and homology for subgroup complexes.

Award-winning monograph of the Ferran Sunyer i Balaguer Prize 2001. Subgroup growth studies the distribution of subgroups of finite index in a group as a function of the index. In the last two decades this topic has developed into one of the most active areas of research in infinite group theory; this book is a systematic and comprehensive account of the substantial theory which has emerged. As well as determining the range of possible 'growth types' for finitely generated groups in general and for groups in particular classes such as linear groups, a main focus of the book is on the tight connection between the subgroup growth of a group and its algebraic structure. A wide range of mathematical disciplines play a significant role in this work: as well as various aspects of infinite group theory, these include finite simple groups and permutation groups, profinite groups, arithmetic groups and Strong Approximation, algebraic and analytic number theory, probability, and p-adic model theory. Relevant aspects of such topics are explained in self-contained 'windows'.

Let $G=\langle G(\mathbb{C}) \rangle$ be a simple algebraic group defined over an algebraically closed field $\mathbb{C}$ of characteristic $p\geq 0$. A subgroup $X\subseteq G$ of $G$ is said to be $G$-completely reducible if, whenever it is contained in a parabolic subgroup of $G$, it is contained in a Levi subgroup of that parabolic. A subgroup $X\subseteq G$ of $G$ is said to be $G$-irreducible if $X$ is in no proper parabolic subgroup of $G$, and $G$-reducible if it is in some proper parabolic of $G$. In this paper, the author considers the case that $G=F_4(\mathbb{K})$. The author finds all conjugacy classes of closed, connected, semisimple $G$-reducible subgroups $X\subseteq G$. Thus he also finds all non-$G$-completely reducible closed, connected, semisimple subgroups of $G$. When $\mathbb{K}$ is connected, closed and simple of rank at least two, he finds all conjugacy classes of $G$-irreducible subgroups $X\subseteq G$. Together with the work of Amende classifying irreducible subgroups of type $A$, this gives a complete classification of the simple subgroups of $G$. The author also uses this classification to find all subgroups of $G=F_4(\mathbb{K})$ which are generated by short root elements of $G$, $\mathbb{K}$, by utilising and extending the results of Liebeck and Seitz.

The Subgroup Structure of the Finite Classical Groups

Let $G=G(\mathbb{K})$ be a simple algebraic group defined over an algebraically closed field with natural module. Let be a closed subgroup of and let be a non-trivial irreducible tensor-indecomposable -restricted rational -module such that the restriction of to is irreducible. In this paper the authors classify the triples of this form, where is a disconnected maximal positive-dimensional closed subgroup of preserving a natural geometric structure on .

The central theme of this monograph is the relation between the structure of a group and the structure of its lattice of subgroups. Since the first papers on this topic have appeared, notably those of BAER and ORE, a large body of literature has grown up around this theory, and it is our aim to give a picture of the present state of this theory. To obtain a systematic treatment of the subject quite a few unpublished results of the author had to be included. On the other hand, it is natural that we could not reproduce every detail and had to treat some parts some what sketchily. We have tried to make this report as self-contained as possible. Accordingly we have given some proofs in considerable detail, though of course it is in the nature of such a report that many proofs have to be omitted or can only be given in outline. Similarly references to the concepts and theorems used are almost exclusively references to standard works like BIRKHOFF [J] and ZASSENHAUS [J]. The author would like to express his sincere gratitude to Professors REINHOLD BAER and DONALD G. HIGMAN for their kindness in giving him many valuable suggestions. His thanks are also due to Dr. NOBORU ITÔ who, during stimulating conversations, contributed many useful ideas. Urbana, May, 1956. M. Suzuki. Contents.

Table of contents

The impetus for current research in pro-$p$-groups comes from four main directions: from new applications in number theory, which continue to be a source of deep and challenging problems; from the traditional problem of classifying finite $p$-groups; from questions arising in infinite group theory; and finally, from the younger subject of 'profinite group theory'. A correspondingly diverse range of mathematical techniques is being successfully applied, leading to new results and pointing to exciting new directions of research. In this work important theoretical developments are carefully presented by leading mathematicians in the field, bringing the reader to the cutting edge of current research. With a systematic emphasis on the construction and examination of many classes of examples, the book presents a clear picture of the rich universe of pro-$p$-groups, in its unity and diversity. Thirty open problems are discussed in the appendix. For graduate students and researchers in group theory, number theory, and algebra, this work will be an indispensable reference text and a rich source of promising avenues for further exploration.

This book classifies the maximal subgroups of the almost simple finite classical groups in dimension up to 12; it also describes the maximal subgroups of the almost simple finite exceptional groups with socle one of $Sz(q)$, $G_2(q)$, $2G_2(q)$ or $3D_4(q)$. Theoretical and computational tools are used throughout, with downloadable Magma code provided. The exposition contains a wealth of information on the structure and action of the geometric groups of classical subgroups, but the reader will also encounter methods for analysing the structure and maximality of almost simple subgroups of almost simple groups. Additionally, this book contains detailed information on using Magma to calculate with representations over number fields and finite fields. Featured within are previously unseen results and over 80 tables describing the maximal subgroups, making this volume an essential reference for researchers. It also functions as a graduate-level textbook on finite simple groups, computational group theory and representation theory.

In this paper, we complete the determination of the maximal subgroups of positive dimension in simple algebraic groups of exceptional type over algebraically closed fields. This follows work of Dynkin, who solved the problem in characteristic zero, and Seitz who did likewise over fields whose characteristic is not too small. A number of consequences are obtained. It follows from the main theorem that a simple algebraic group over an algebraically closed field has only finitely many conjugacy classes of maximal subgroups of positive dimension. It also follows that the maximal subgroups of sufficiently large order in finite exceptional groups of Lie type are known.

Research in Personnel and Human Resources Management is designed to promote theory and research on important substantive and methodological topics in the field of
human resources management.

The study of finite groups factorised as a product of two or more subgroups has become a subject of great interest during the last years with applications not only in group theory, but also in other areas like cryptography and coding theory. It has experienced a big impulse with the introduction of some permutability conditions. The aim of this book is to gather, order, and examine part of this material, including the latest advances made, give some new approach to some topics, and present some new subjects of research in the theory of finite factorised groups.

It is a great satisfaction for a mathematician to witness the growth and expansion of a theory in which he has taken some part during its early years. When H. Weyl coined the words "classical groups", foremost in his mind were their connections with invariant theory, which his famous book helped to revive. Although his approach in that book was deliberately algebraic, his interest in these groups directly derived from his pioneering study of the special case in which the scalars are real or complex numbers, where for the first time he injected Topology into Lie theory. But ever since the definition of Lie groups, the analogy between simple classical groups over finite fields and simple classical groups over IR or C had been observed, even if the concept of "simplicity" was not quite the same in both cases. With the discovery of the exceptional simple complex Lie algebras by Killing and E. Cartan, it was natural to look for corresponding groups over finite fields, and already around 1900 this was done by Dickson for the exceptional Lie algebras G and E. However, a deep reason for this 2 6 parallelism was missing, and it is only Chevalley who, in 1955 and 1961, discovered that to each complex Lie algebra corresponds, by a uniform process, a group scheme (fj over the ring Z of integers, from which, for any field K, could be derived a group (fj(K)

Abstract - Let $G$ be a simple algebraic group of exceptional type over an algebraically closed field of characteristic $p$. Under some mild restrictions on $p$, we classify all conjugacy classes of closed connected subgroups $X$ of $G$; for each such class of subgroups, we also determine the connected centralizer and the composition factors in the action on the Lie algebra $L(G)$ of $G$. Moreover, we show that $\mathfrak{L}(C_G(X))=C_(\mathfrak{L}(G))\mathfrak{L}(X)$ for each subgroup $X$. These results build upon recent work of Liebeck and Seitz, who have provided similar detailed information for closed connected subgroups of rank at least 22. In addition, for any such subgroup $X$ we identify the unipotent class $\mathfrak{L}(C_G(X))$ of $G$ and the labelled diagram of $X$, and we establish the existence of a subgroup $X$ in $\mathfrak{L}(C_G(X))$ of order $p$. We show that in almost all cases the labelled diagram of the class $\mathfrak{L}(C_G(X))$ may easily be obtained from that of $X$; furthermore, if $\mathfrak{L}(C_G(X))$ is a conjugacy class of elements of order $p$, we establish the existence of a subgroup $X$ in $\mathfrak{L}(C_G(X))$ and having the same labelled diagram as $\mathfrak{L}(C_G(X))$.

Let be a simple classical algebraic group over an algebraically closed field of characteristic with natural module . Let be a closed subgroup of and let be a nontrivial -restricted irreducible tensor indecomposable rational -module such that the restriction of to is irreducible. In this paper the authors classify the triples of this form, where and is a disconnected almost simple positive-dimensional closed subgroup of acting irreducibly on . Moreover, by combining this result with earlier work, they complete the classification of the irreducible triples where is a simple algebraic group over , and is a maximal closed subgroup of positive dimension.

This paper is a contribution to the study of the subgroup structure of excep-tional algebraic groups over algebraically closed ?elds of arbitrary characteristic. Following Serre, a closed subgroup of a semisimple algebraic group $G$ is called irreducible if it lies in no proper parabolic subgroup of $G$. In this paper we com-plete the classi?cation of irreducible connected subgroups of exceptional algebraic groups, providing more explicit information for the conjugacy classes of such subgroups. Many consequences of this classi?cation are also given. These include results concerning the representations of such subgroups on various $G$-modules: for example, the conjugacy classes of irreducible connected subgroups are determined by their composition factors on the adjoint module of $G$, with one exception. A result of Liebeck and Testerman shows that each irreducible connected sub-group $X$ of $G$ has only ?nitely many overgroups and hence the overgroups of $X$ form a lattice. We provide tables that give representatives of each conjugacy class of connected overgroups within this lattice structure. We use this to prove results concerning the subgroup structure of $G$: for example, when the characteristic is 2, there exists a maximal connected subgroup of $G$ containing a conjugate of every irreducible subgroup A1 of $G$.

The authors address the classical problem of determining finite primitive permutation groups $G$ with a regular subgroup $B$. The main theorem solves the problem completely under the assumption that $G$ is almost simple. While there are many examples of regular subgroups of small degrees, the list is rather short (just four infinite families) if the degree is assumed to be large enough, for example at least 30! Another result determines all primitive groups having a regular subgroup which is almost simple. This has an application to the theory of Cayley graphs of simple groups.

This volume is an outcome of the International Conference on Algebra in celebration of the 70th birthday of Professor Shum Kar-Ping which was held in Gadjah Mada University on 7Co10 October 2010. As a consequence of the wide coverage of his research interest and work, it presents 54 research papers, all original and refereed, describing the latest research and development, and addressing a variety of issues and methods in semigroups, groups, rings and modules, lattices and Hopf Algebra. The book also provides five well-written expository survey articles which feature the structure of finite groups by A Ballester-Bolinches, R Esteban-Romero, and Yangming Li; new results of Grdner-Shirshov basis by L A Bokut, Yuqun Chen, and K P Shum; polygroups and their properties by B Davvaz; main results on abstract characterizations of algebras of n-place functions obtained in the last 40 years by Wieslaw A Dudek and Valentin S Trokhimenko; Inverse semigroups and their generalizations by X M Ren and K P Shum. Recent work
on cones of metrics and combinatorics done by M M Deza et al. is included."

The author extends results of McLaughlin and Kantor on overgroups of long root subgroups and long root elements in finite classical groups. In particular he determines the maximal subgroups of this form. He also determines the maximal overgroups of short root subgroups in finite classical groups and the maximal overgroups in finite orthogonal groups of c-root subgroups.

Originating from a summer school taught by the authors, this concise treatment includes many of the main results in the area. An introductory chapter describes the fundamental results on linear algebraic groups, culminating in the classification of semisimple groups. The second chapter introduces more specialized topics in the subgroup structure of semisimple groups and describes the classification of the maximal subgroups of the simple algebraic groups. The authors then systematically develop the subgroup structure of finite groups of Lie type as a consequence of the structural results on algebraic groups. This approach will help students to understand the relationship between these two classes of groups. The book covers many topics that are central to the subject, but missing from existing textbooks. The authors provide numerous instructive exercises and examples for those who are learning the subject as well as more advanced topics for research students working in related areas.

With the classification of the finite simple groups complete, much work has gone into the study of maximal subgroups of almost simple groups. In this volume the authors investigate the maximal subgroups of the finite classical groups and present research into these groups as well as proving many new results. In particular, the authors develop a unified treatment of the theory of the ‘geometric subgroups’ of the classical groups, introduced by Aschbacher, and they answer the questions of maximality and conjugacy and obtain the precise shapes of these groups. Both authors are experts in the field and the book will be of considerable value not only to group theorists, but also to combinatorialists and geometers interested in these techniques and results. Graduate students will find it a very readable introduction to the topic and it will bring them to the very forefront of research in group theory.

This is the sixth volume of a comprehensive and elementary treatment of finite group theory. This volume contains many hundreds of original exercises (including solutions for the more difficult ones) and an extended list of about 1000 open problems. The current book is based on Volumes 1–5 and it is suitable for researchers and graduate students working in group theory.

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